Numerical Question Bank
MATHEMATICS (BE-301)
Semester: 3RD

INTRUCTIONS. 1. All questions with their solution are submitted till 27 October 2014.

UNIT-I

1. Obtain the Fourier series for $f(x)=e^{-x}$ in the interval $0<x<2\pi$
2. Find a Fourier series represented $x-x^2$ from $x=-\pi$ to $x=\pi$. Hence show that
$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots\ldots\ \ldots\ldots=\frac{\pi^2}{12}$$
3. Obtain the Fourier series for the function $f(x)=x^2 \quad ,-\pi\le x\le\pi$
$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots\ldots=\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$$
4. Find the Fourier series to represented the function , if
$$f(x)=\begin{cases}-\pi & , -\pi<x<0\\ x & , \quad 0<x<\pi\end{cases}$$
5. Find the Fourier series to represented the function
$$f(x)=\begin{cases}0 & , -\pi<x<0\\ \sin x & , \quad 0<x<\pi\end{cases}$$
Hence show that $\frac{1}{1.3}+\frac{1}{3.5}+\ldots\ldots\ldots=\frac{1}{2}$
6. Find the Fourier series represented the function $f(x)=x\sin x$, $0<x<2\pi$
7. Find the Fourier series of the function $f(x)=x^2 \quad ,-\pi<\mathrm{x}<\pi$
8. Find the Fourier series of the function $f(x)=\begin{cases}\pi x & , 0\le x\le 1\\ \pi(2-x) & , 1\le x\le 2\end{cases}$
9. Find cosine Fourier series of $f(x)=|x|\,\text{in}\,(0,l)$.
10. Find the half range sine Fourier series for the function $f(x)=x$ in $(0,\pi)$.
11. Express $f(x)=x$ as cosine series in half range series in $0<x<2$
12. Express $f(x)=x$ as a half range sine series in $0<x<2$
13. Find the Fourier transform of $f(x)\begin{cases}1 & , \text{for}\,|x|<1\\ 0 & , \text{for}\,|x|>1\end{cases}$

Hence evolution $\int_0^{\infty}\frac{\sin x}{x}dx$

14. Find Fourier sine transform of : $f(x)=\begin{cases} x & \text{if } 0<x<1 \\ 2-x & \text{if } 1<x<2 \\ 0 & \text{if } x>2 \end{cases}$

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15Find sine transform of $\frac{e^{-ax}}{x}$

15. Find the Fourier cosine transform of e^{-x^2}.

16. Find Fourier cosine transform of $f(x)=e^{-x}$

And show that $\int_0^\infty \frac{\cos sx}{1+s^2}ds=\frac{\pi}{2}e^{-x}$

17. Find Fourier sine transform of $f(x)=e^{-x}$

And prove that $\int_0^\infty \frac{x}{x^2+1}\sin mx dx=\frac{\pi}{2}e^{-m}$

18. Using Fourier integral , prove that

$$e^{-ax}=\frac{2a}{\pi}\int_0^\infty \frac{\cos\lambda x}{\lambda^2+a^2}d\lambda$$

Unit –II

19. Find the Laplace transform of (i) $\sin 3t\sin 4t$ (ii) $\sin^3 2t$ (iii) $\sqrt{\sin t}$

20. Find the Laplace transform of (i) $e^{2t}\sin t$ (ii) $e^{-t}(3\sinh 2t-5\cosh 2t)$

21. Find Laplace transform of $\frac{\cos\sqrt{t}}{\sqrt{t}}$ if $L\{\sin\sqrt{t}\}=\frac{\sqrt{\pi}e^{-\frac{1}{4s}}}{2s^{\frac{3}{2}}}$

22. Find Laplace transform of (i) $t\sin at$ (ii) $t^2\sin at$ (iii) $t\cos at$ (iv) $te^{-2t}\sin 2t$

23. Find Laplace transform of the following functions :

(i) $\frac{\sin t}{t}$ (ii) $\frac{\cos at-\cos bt}{t}$ (iii) $\frac{1-\cos 2t}{t}$

25. Find the Laplace transform of (i) $\int_0^t \frac{\sin t}{t}$ dt (ii) $\int_0^t \frac{e^t\sin t}{t}dt$

26. Find the inverse Laplace transform of (i) $\frac{s}{(s^2+1)(s^2+4)}$ (ii) $\frac{s^3}{s^4-a^4}$ (iii)

$\frac{s^2+6}{(s^2+1)(s^2+4)}$

27. Find the inverse Laplace transform of (i) $\log\left(\frac{s+1}{s-1}\right)$ (ii) $\log\left(\frac{s^2-1}{s^2}\right)$ (iii) $\log\left(\frac{s(s+1)}{s^2+4}\right)$

28. Apply convolution theorem to evaluate: $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$

29. Apply convolution theorem to evaluate: $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$

30. Apply convolution theorem to evaluate: $L^{-1}\left\{\frac{1}{(s^2+9)(s^2+3)}\right\}$

31. Apply convolution theorem to evaluate: $L^{-1}\left\{\frac{1}{s(s^2-a^2)}\right\}$

32. Solve the differential equation
$y''(t)-3y'(t)+2y(t)=4t+e^{3t}$ when $y(0)=1$ and $y'(0)=-1$

33. Solve the equation $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+5x=e^{-t}\sin t$, $x(0)=0$, $x'(0)=1$

34. Use Laplace transform method to solve :

$$\frac{d^2x}{dt^2}-2\frac{dx}{dt}+x=e^t \text{ , with } x=2, \frac{dx}{dt}=-1 \quad at \ t=0$$

35. Solve $(D^2+9)y=\cos 2t$ if $y(0)=1$, $y\left(\frac{\pi}{2}\right)=-1$

36. Solve the equation by Laplace transform

$$\frac{dy}{dt}+2y+\int_0^t y\,dt=t \text{ , with } y(0)=1$$

Unit-III

1. Solve the following differential equation :

$$x\frac{d^2y}{dx^2}-(2x-1)\frac{dy}{dx}+(x-1)y=0$$

2. Solve, $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0,$ given that $x+\frac{1}{x}$ is one integral.

3. Solve the equation $x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+4y=0,$ when $y(0)=4$ and $\frac{dy}{dx}=13$ at $x=1$

4. Solve $x\frac{d^2y}{dx^2}-2(x+1)\frac{dy}{dx}+(x+2)y=(x-2)e^x$

5. Solve $x^2\frac{d^2y}{dx^2}-2(x^2+x)\frac{dy}{dx}+(x^2+2x+2)y=0$

6. Solve $y''-\frac{2}{x}y'+\left(1+\frac{2}{x^2}\right)y=xe^x$

7. Solve $\frac{d^2y}{dx^2}-2\tan x\frac{dy}{dx}+5y=\sec xe^x$

8. Solve $\cos x\frac{d^2y}{dx^2}+\sin x\frac{dy}{dx}-2\cos^3 x\, y=2\cos^5 x$

9. Solve $x\frac{d^2y}{dx^2}-\frac{dy}{dx}-4x^3y=8x^3\sin(x^2)$

10. Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2}+n^2y=\sec nx$$

11. Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2}+y=\operatorname{cosec} x$$

12. Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2}+4y=4\tan 2x$$

13. Solve by using variation of parameters

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}=e^x\sin x$$

14. Solve by using variation of parameters

$$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=\frac{e^{3x}}{x^2}$$

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15. Solve in series of the following equation :

$$(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$$

16. Solve in series $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=0$

17. Solve in series of the following equation

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$$

(Which is known is Legendre's equation)

18. Solve in series of the following equation :

$$2x^2\frac{d^2y}{dx^2}+\left(2x^2-x\right)\frac{dy}{dx}+y=0$$

19. Solve in series of the following equation :

$$\left(2x+x^3\right)\frac{d^2y}{dx^2}-\frac{dy}{dx}-6xy=0$$

20. Solve in series $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^2-n^2\right)y=0$

(which is Bessel's equation)

21. Obtain the series solution of the equation :

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^2-4\right)y=0$$

Unit -IV

22. From the partial differential equation by eliminating the arbitrary function from the following

(i) $z=f\left(x^2-y^2\right)$ (ii) $z=f(x+iy)+g(x-iy)$

(iii) $z=y^2+2yf\left(\frac{1}{x}+\log y\right)$

23. From the partial differential equation from the following

(i) $f(x^2+y^2, z-ax)=0$ (ii) $f(x+y+z, x^2+y^2+z^2)=0$

24. Solve $y^2zp+x^2zq=xy^2$

25. Solve $\left(z^2-2xy-y^2\right)p+(xy+zx)q=xy-zx$

26. Solve $y^2p-xyq=x(z-2y)$

27. Solve $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$

28. Solve $(x^2-yz)p+\left(y^2-zx\right)q=z^2-xy$

29. Solve $x^2p+y^2q=(x+y)z$

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30. Solve $x(y-z)p+y(z-x)q=z(x-y)$

31. Solve $pz-qz=z^2+(x+y)^2$

32. Solve $x^2p^2+y^2q^2=z^2$

33. Solve $z=p^2+q^2$

34. Find the complete and singular solution of $z^2\left(p^2z^2+q^2\right)=1$

35. Solve $z^2\left(p^2+q^2\right)=x^2+y^2$

36. Solve $\left(p^2+q^2\right)y=qz$ by using charpits method.

37. Solve $z = px + qy + p^2 + q^2$ by charpit's method.

38. Solve $pxy + pq + qy = yz$ by using charpit's method.

39. Solve the equation by charpit's method

$$2(z + xp + yq) = yp^2$$

40. Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ by the method of separation of variable where $u(x,0) = 6e^{-3x}$.

41. Solve by the method of separation of variable $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$

42. Solve the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial t^2}$ subject to the conditions

$$y(0,t) = 0, y(l,t) = 0, y(x,0) = y_0 \sin\frac{\pi x}{l} \text{ and } \left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$$

43. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is relased from rest from this position , Find the displacement $y(x,t)$.

Unit-V (VECTOR CALCULUS)

44. Find the unit normal to the surface

$$x^4 - 3xyz + z^2 + 1 = 0 \text{ at the point (1, 1, 1).}$$

45. Evaluate $\iint_S \vec{F}.\hat{n}\, ds$, where $\vec{F} = (x + y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$ and S is the surface of the Plane $2x + y + 2z = 6$ in the first octant.

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46. Find $div\vec{F}$ and $curl\vec{F}$, when $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$.

47. Verify Stoke's theorem for $\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$, taken around the rectangle in the plane $z = 0$ and bounded by the lines $x = 0, y = 0, x = a, y = b$.