

# Numerical Question Bank

## MATHEMATICS (BE-301)

### Semester: 3<sup>RD</sup>

**INSTRUCTIONS. 1. All questions with their solution are submitted till 27 October 2014.**

#### UNIT-I

- Obtain the Fourier series for  $f(x) = e^{-x}$  in the interval  $0 < x < 2\pi$
- Find a Fourier series represented  $x - x^2$  from  $x = -\pi$  to  $x = \pi$ . Hence show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$$

- Obtain the Fourier series for the function  $f(x) = x^2$ ,  $-\pi \leq x \leq \pi$

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} + \dots = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

- Find the Fourier series to represented the function, if

$$f(x) = \begin{cases} -\pi & , -\pi < x < 0 \\ x & , 0 < x < \pi \end{cases}$$

- Find the Fourier series to represented the function

$$f(x) = \begin{cases} 0 & , -\pi < x < 0 \\ \sin x & , 0 < x < \pi \end{cases}$$

Hence show that  $\frac{1}{1.3} + \frac{1}{3.5} + \dots = \frac{1}{2}$

- Find the Fourier series represented the function  $f(x) = x \sin x$ ,  $0 < x < 2\pi$
- Find the Fourier series of the function  $f(x) = x^2$ ,  $-\pi < x < \pi$
- Find the Fourier series of the function  $f(x) = \begin{cases} \pi x & , 0 \leq x \leq 1 \\ \pi(2-x) & , 1 \leq x \leq 2 \end{cases}$
- Find cosine Fourier series of  $f(x) = |x|$  in  $(0, l)$ .
- Find the half range sine Fourier series for the function  $f(x) = x$  in  $(0, \pi)$ .
- Express  $f(x) = x$  as cosine series in half range series in  $0 < x < 2$
- Express  $f(x) = x$  as a half range sine series in  $0 < x < 2$
- Find the Fourier transform of  $f(x) = \begin{cases} 1 & , \text{for } |x| < 1 \\ 0 & , \text{for } |x| > 1 \end{cases}$

Hence evolution  $\int_0^{\infty} \frac{\sin x}{x} dx$

14. Find Fourier sine transform of :  $f(x) = \begin{cases} x & \text{if } 0 < x < 1 \\ 2 - x & \text{if } 1 < x < 2 \\ 0 & \text{if } x > 2 \end{cases}$

**ENGG.MATHEMATICS-II (BY Alkesh Kumar Dhakde-09407270889)**

15. Find sine transform of  $\frac{e^{-ax}}{x}$

15. Find the Fourier cosine transform of  $e^{-x^2}$ .

16. Find Fourier cosine transform of  $f(x) = e^{-x}$

And show that  $\int_0^{\infty} \frac{\cos sx}{1+s^2} ds = \frac{\pi}{2} e^{-x}$

17. Find Fourier sine transform of  $f(x) = e^{-x}$

And prove that  $\int_0^{\infty} \frac{x}{x^2+1} \sin mx dx = \frac{\pi}{2} e^{-m}$

18. Using Fourier integral, prove that

$$e^{-ax} = \frac{2a}{\pi} \int_0^{\infty} \frac{\cos \lambda x}{\lambda^2 + a^2} d\lambda$$

**Unit –II**

19. Find the Laplace transform of (i)  $\sin 3t \sin 4t$  (ii)  $\sin^3 2t$  (iii)  $\sqrt{\sin t}$

20. Find the Laplace transform of (i)  $e^{2t} \sin t$  (ii)  $e^{-t}(3 \sinh 2t - 5 \cosh 2t)$

21. Find Laplace transform of  $\frac{\cos \sqrt{t}}{\sqrt{t}}$  if  $L\{\sin \sqrt{t}\} = \frac{\sqrt{\pi} e^{-\frac{1}{4s}}}{2s^{\frac{3}{2}}}$

22. Find Laplace transform of (i)  $t \sin at$  (ii)  $t^2 \sin at$  (iii)  $t \cos at$  (iv)  $te^{-2t} \sin 2t$

23. Find Laplace transform of the following functions :

(i)  $\frac{\sin t}{t}$  (ii)  $\frac{\cos at - \cos bt}{t}$  (iii)  $\frac{1 - \cos 2t}{t}$

25. Find the Laplace transform of (i)  $\int_0^t \frac{\sin t}{t} dt$  (ii)  $\int_0^t \frac{e^t \sin t}{t} dt$

26. Find the inverse Laplace transform of (i)  $\frac{s}{(s^2+1)(s^2+4)}$  (ii)  $\frac{s^3}{s^4 - a^4}$  (iii)

$$\frac{s^2 + 6}{(s^2 + 1)(s^2 + 4)}$$

27. Find the inverse Laplace transform of (i)  $\log\left(\frac{s+1}{s-1}\right)$  (ii)  $\log\left(\frac{s^2-1}{s^2}\right)$  (iii)

$$\log\left(\frac{s(s+1)}{s^2+4}\right)$$

28. Apply convolution theorem to evaluate:  $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$

29. Apply convolution theorem to evaluate:  $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$

30. Apply convolution theorem to evaluate:  $L^{-1}\left\{\frac{1}{(s^2+9)(s^2+3)}\right\}$

31. Apply convolution theorem to evaluate:  $L^{-1}\left\{\frac{1}{s(s^2-a^2)}\right\}$

32. Solve the differential equation

$$y''(t) - 3y'(t) + 2y(t) = 4t + e^{3t} \quad \text{when } y(0) = 1 \text{ and } y'(0) = -1$$

33. Solve the equation  $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t} \sin t$ ,  $x(0) = 0$ ,  $x'(0) = 1$

34. Use Laplace transform method to solve :

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t, \text{ with } x = 2, \frac{dx}{dt} = -1 \text{ at } t = 0$$

35. Solve  $(D^2 + 9)y = \cos 2t$  if  $y(0) = 1$ ,  $y\left(\frac{\pi}{2}\right) = -1$

36. Solve the equation by Laplace transform

$$\frac{dy}{dt} + 2y + \int_0^t y dt = t, \text{ with } y(0) = 1$$

1. Solve the following differential equation :

$$x \frac{d^2 y}{dx^2} - (2x-1) \frac{dy}{dx} + (x-1)y = 0$$

2. Solve,  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = 0$ , given that  $x + \frac{1}{x}$  is one integral.

3. Solve the equation  $x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + 4y = 0$ , when  $y(0) = 4$  and  $\frac{dy}{dx} = 13$  at  $x = 1$

4. Solve  $x \frac{d^2 y}{dx^2} - 2(x+1) \frac{dy}{dx} + (x+2)y = (x-2)e^x$

5. Solve  $x^2 \frac{d^2 y}{dx^2} - 2(x^2 + x) \frac{dy}{dx} + (x^2 + 2x + 2)y = 0$

6. Solve  $y'' - \frac{2}{x} y' + \left(1 + \frac{2}{x^2}\right)y = xe^x$

7. Solve  $\frac{d^2 y}{dx^2} - 2 \tan x \frac{dy}{dx} + 5y = \sec x e^x$

8. Solve  $\cos x \frac{d^2 y}{dx^2} + \sin x \frac{dy}{dx} - 2 \cos^3 x y = 2 \cos^5 x$

9. Solve  $x \frac{d^2 y}{dx^2} - \frac{dy}{dx} - 4x^3 y = 8x^3 \sin(x^2)$

10. Solve by the method of variation of parameters

$$\frac{d^2 y}{dx^2} + n^2 y = \sec nx$$

11. Solve by the method of variation of parameters

$$\frac{d^2 y}{dx^2} + y = \operatorname{cosec} x$$

12. Solve by the method of variation of parameters

$$\frac{d^2 y}{dx^2} + 4y = 4 \tan 2x$$

13. Solve by using variation of parameters

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} = e^x \sin x$$

14. Solve by using variation of parameters

$$\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$$

### ENGG.MATHEMATICS-II (BY Alkesh Kumar Dhakde-09407270889)

15. Solve in series of the following equation :

$$(1+x^2) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = 0$$

16. Solve in series  $(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + 4y = 0$

17. Solve in series of the following equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$

(Which is known as Legendre's equation)

18. Solve in series of the following equation :

$$2x^2\frac{d^2y}{dx^2} + (2x^2 - x)\frac{dy}{dx} + y = 0$$

19. Solve in series of the following equation :

$$(2x + x^3)\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6xy = 0$$

20. Solve in series  $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$

( which is Bessel's equation )

21. Obtain the series solution of the equation :

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - 4)y = 0$$

#### Unit -IV

22. From the partial differential equation by eliminating the arbitrary function from the following

(i)  $z = f(x^2 - y^2)$       (ii)  $z = f(x + iy) + g(x - iy)$

(iii)  $z = y^2 + 2yf\left(\frac{1}{x} + \log y\right)$

23. From the partial differential equation from the following

(i)  $f(x^2 + y^2, z - ax) = 0$       (ii)  $f(x + y + z, x^2 + y^2 + z^2) = 0$

24. Solve  $y^2zp + x^2zq = xy^2$

25. Solve  $(z^2 - 2xy - y^2)p + (xy + zx)q = xy - zx$

26. Solve  $y^2p - xyq = x(z - 2y)$

27. Solve  $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$

28. Solve  $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

29. Solve  $x^2p + y^2q = (x + y)z$

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30. Solve  $x(y - z)p + y(z - x)q = z(x - y)$

31. Solve  $pz - qz = z^2 + (x + y)^2$

32. Solve  $x^2p^2 + y^2q^2 = z^2$

33. Solve  $z = p^2 + q^2$

34. Find the complete and singular solution of  $z^2(p^2z^2 + q^2) = 1$

35. Solve  $z^2(p^2 + q^2) = x^2 + y^2$

36. Solve  $(p^2 + q^2)y = qz$  by using charpits method.

37. Solve  $z = px + qy + p^2 + q^2$  by charpit's method.

38. Solve  $pxy + pq + qy = yz$  by using charpit's method.

39. Solve the equation by charpit's method

$$2(z + xp + yq) = yp^2$$

40. Solve  $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$  by the method of separation of variable where  $u(x,0) = 6e^{-3x}$ .

41. Solve by the method of separation of variable  $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$

42. Solve the wave equation  $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$  subject to the conditions

$$y(0,t) = 0, y(l,t) = 0, y(x,0) = y_0 \sin \frac{\pi x}{l} \text{ and } \left( \frac{\partial y}{\partial t} \right)_{t=0} = 0$$

43. A tightly stretched string with fixed end points  $x=0$  and  $x=l$  is initially in a position given by  $y = y_0 \sin^3 \left( \frac{\pi x}{l} \right)$ . If it is released from rest from this position, Find the displacement  $y(x,t)$ .

### **Unit-V (VECTOR CALCULUS)**

44. Find the unit normal to the surface

$$x^4 - 3xyz + z^2 + 1 = 0 \text{ at the point } (1, 1, 1).$$

45. Evaluate  $\iint_S \vec{F} \cdot \hat{n} \, ds$ , where  $\vec{F} = (x + y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$  and S is the surface of the Plane  $2x + y + 2z = 6$  in the first octant.

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46. Find  $\text{div } \vec{F}$  and  $\text{curl } \vec{F}$ , when  $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ .

47. Verify Stoke's theorem for  $\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$ , taken around the rectangle in the plane  $z = 0$  and bounded by the lines  $x = 0, y = 0, x = a, y = b$ .